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Rotational Motion and Equilibrium

Physics Rapid Learning Series

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Learning Objectives

By completing this tutorial, you will:

- **Describe the kinematics of rotational motion.**
- **Understand the concept of torque.**
- **Understand the concept of moment of inertia**
- **Apply the concepts of torque and moment of inertia to rotational motion and equilibrium.**
- **Extend your knowledge of momentum to the rotational variety.**

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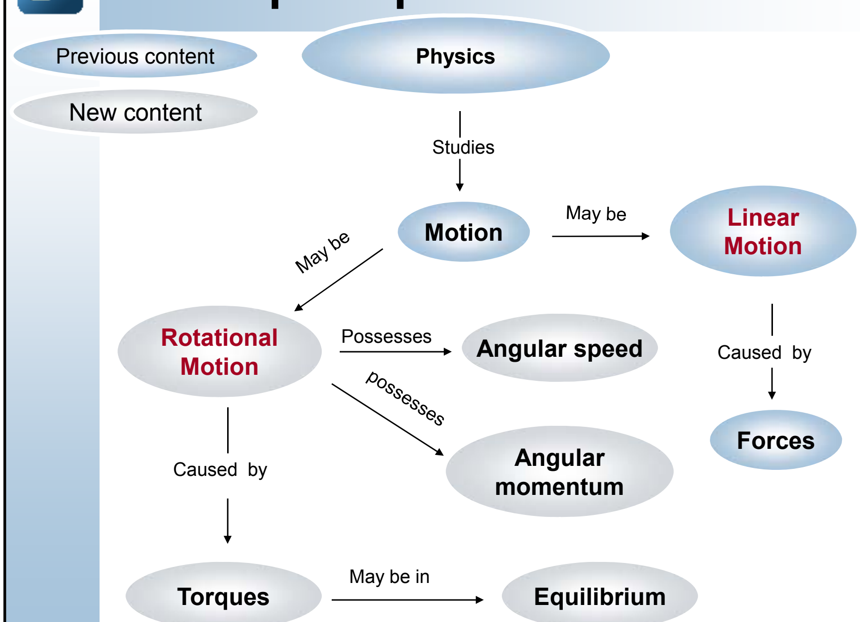
Concept Map



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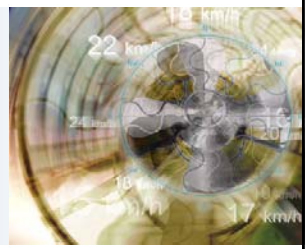
Kinematics of Rotational Motion



Our previous knowledge of kinematics and motion in a line can be extended to describe rotational motion.

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Angular Analogs

When linear distance, d, was measured, we used meters or similar metric unit.

Now we must use radians to measure angular or rotational distance, θ.

Thus a radian will be analogous to a meter, and angle will be analogous to distance for this tutorial.

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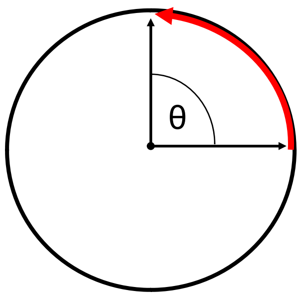
The Radian

Angle, radians

Linear arc distance traveled, m



$$\theta = \frac{s}{r}$$

Radius of circular path, m

Note that meters and meters cancels out. Thus a radian has no other dimensional units.

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Radians and Circles

A useful fact to remember is that one complete circle equals 2π radians.

If a wheel turns 200 rpm, revolutions per minute, how many radians per second is that?

$$200\,\frac{\text{rev}}{\text{min}} \times \frac{2\pi\,\text{rad}}{1\,\text{rev}} \times \frac{1\,\text{min}}{60\,\text{s}} = 20.9\ \text{rad/sec}$$

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Angular Velocity

Just as there is a rotational equivalent to linear distance, there is one for linear speed.

Angular speed, rad/s

Angle, rad

$$\omega = \frac{\Delta\theta}{\Delta t}$$

Time,s

This is comparable to our linear speed formula:

V = d/t

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Angular Acceleration

Just as there is a rotational equivalent to linear velocity, there is one for linear acceleration.

Angular acceleration, rad/s²

Change in angular speed, radians/s

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

Time,s

This is comparable to our linear acceleration formula: a = Δv/Δt

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Additional Angular Formulas

We already have two definitions or formulas for angular motion that are similar to previous linear ones.

Linear Motion	Angular Motion
$v = \frac{d}{t}$	$\omega = \frac{\Delta\theta}{\Delta t}$
$a = \frac{\Delta v}{\Delta t}$	$\alpha = \frac{\Delta\omega}{\Delta t}$
$d = v_i t + at^2/2$	$\theta = \omega_i t + \alpha t^2/2$
$v_f^2 = v_i^2 + 2ad$	$\boldsymbol{\omega_f^2 = \omega_i^2 + 2\alpha\theta}$

There are corresponding formulas for our other linear relationships too.

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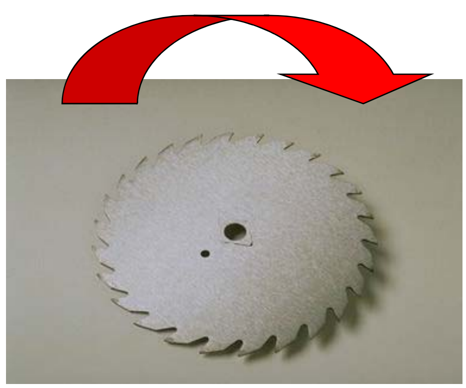
Angular Motion Example - Question

The blade in an electric circular saw starts at rest. When initiated, the blade reaches an angular velocity of 90 rev/s in 250 revolutions. What is the angular acceleration of the blade?



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Angular Motion Example - Solution

Given quantities:

$\omega_i = 0$ rad/s $\omega_f = 90$ rev/s $\theta = 250$ rev

Unknown quantity:

angular acceleration, α

Likely formula to use:

$\omega_f^2 = \omega_i^2 + 2\alpha\theta$

Rearranging for unknown quantity:

$$\alpha = \frac{\omega_f^2 - \omega_i^2}{2\theta}$$

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Angular Motion Example - Calculation

First, units must be converted:

$$\omega_f = 90\frac{\text{rev}}{\text{s}} \times \frac{2\pi\ \text{rad}}{1\text{rev}} = 180\pi\ \text{rad/s}$$

$$\theta = 250\text{rev} \times \frac{2\pi\ \text{rad}}{1\text{rev}} = 500\pi\ \text{rad}$$

Next, substitute into our rearranged formula:

$$\alpha = \frac{\omega_f^2 - \omega_i^2}{2\theta}$$

$$\alpha = \frac{(180\pi\ \text{rad/s})^2 - (0\pi\ \text{rad/s})^2}{2(500\pi\ \text{rad})} = 102\ \text{rad/s}^2$$

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Torque

Just as force relates to linear movement, torque has an analogous relationship to rotation.

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Torque Formula

Just as force makes objects move in a line, torque makes object rotate.

$$T = F\,l$$

- **Torque, mN** (T)
- **Force applied, N** (F)
- **Lever arm distance, m** (l)

The notation can be a bit confusing:

Don't confuse T for torque with T for time period.
Don't confuse *l* for lever arm with a number 1.

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Torque Units

As the formula implies, torque has typical metric units of N·m.

You may notice this is very similar to the unit of Joules for work/energy which may be written as N·m.

To avoid confusion, the unit of torque is often written reversed: m·N

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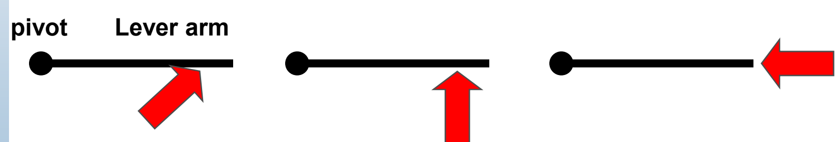
Direction of Force

When calculating torque, only the force that is perpendicular, ⊥ , to the lever arm is considered.

Pushing along, or parallel, to the lever arm doesn't produce any torque!



Some torque

More torque since all force is ⊥ to lever arm.

No torque since no force is ⊥ to lever arm.

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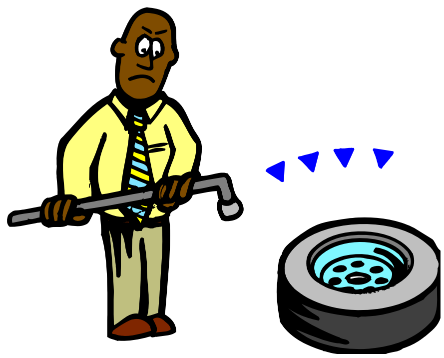
Increasing Torque

When a lot of torque is required to tighten or loosen something, you can either increase your force applied, or increase the lever arm.

This is why many tools have large lever arms or handles to held accomplish the job:



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Torque Wrench

Often, a bolt or other mechanical fastener must be tightened a certain amount. Too loose and the object falls off, too tight and the bolt may snap! To accomplish this, a torque wrench measures exactly that! There is no guess work involved then.

For the particular lever arm, the dial reads the amount of torque applied.

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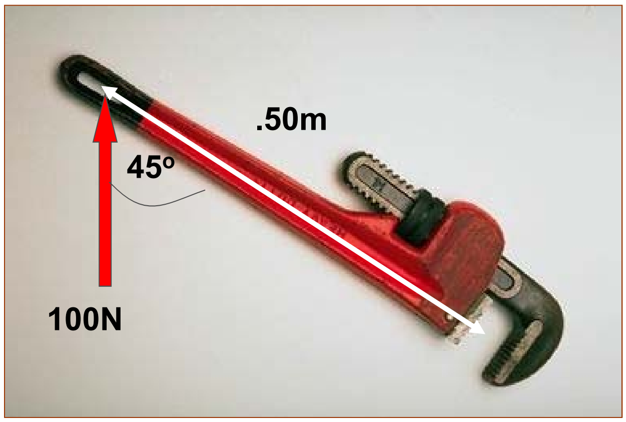
Torque Example - Question

If you apply a 100 N force to the end of the wrench as shown, how much torque are you applying?



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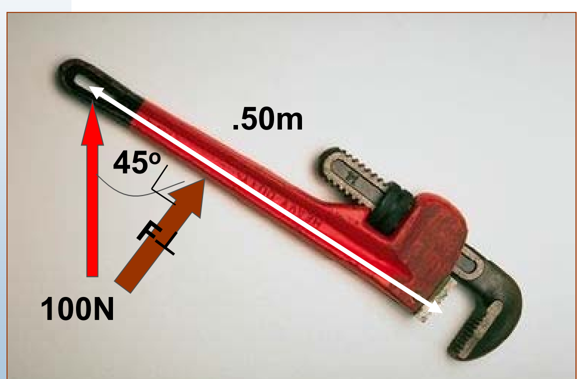
Torque Example - Solution

Remember that we're only using the component of the force that is perpendicular to the lever arm:



$\tau = l\,F$

$\sin\theta = \frac{opp}{hyp} = \frac{F_{\perp}}{hyp}$

$\tau = l\,F\sin\theta$

$\tau = .50\text{m}\ 100\text{N}\ \sin 45^{\circ}$

$\tau = 36\ \text{m}\cdot\text{N}$

This vector represents the component of the applied force that is perpendicular to the lever arm

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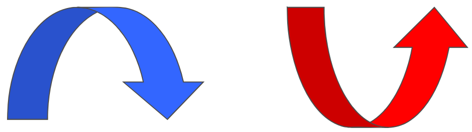
Torque Direction

Torque can have two directions:



Clockwise (CW)

Counterclockwise (CCW)

When there is an unbalanced torque, rotation is caused. This is analogous to an unbalanced force causing acceleration.

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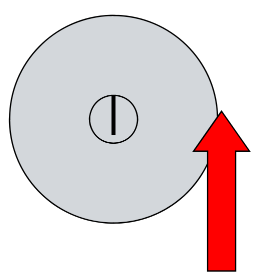
Application of Torque

You've probably experienced torque in the most common of circumstances. For example, every time you open a door, you apply a torque to the handle or knob. The knob exists because it supplies a larger lever arm for you to apply more torque to the door mechanism.

Close-up of knob

Force applied, then rotation occurs.



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Moment of Inertia

Also called rotational inertia, this is the spinning counterpart to linear inertia that you are already familiar with.

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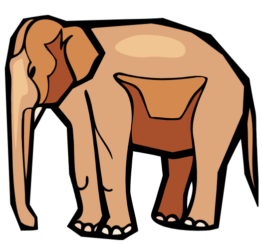
Linear Inertia Review

Linear inertia says that an object moving in a straight line wants to continue moving in a straight line, until acted upon by a force.

Also, an object at rest wants to stay at rest.

Linear inertia is dependent upon mass.



Large amount of inertia!

Small amount of inertia!

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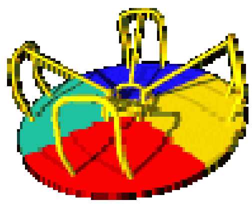
Moment of Inertia

Moment of inertia says that an object rotating about an axis wants to continue rotating until acted on by a force, or torque.



Moment of inertia, or rotational inertia depends on the distribution of the mass in the rotating object.

Although mass is important, so is the location of that mass.

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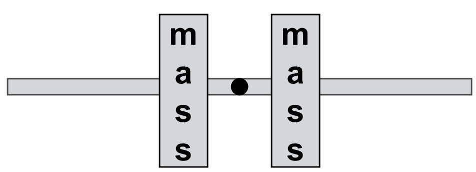
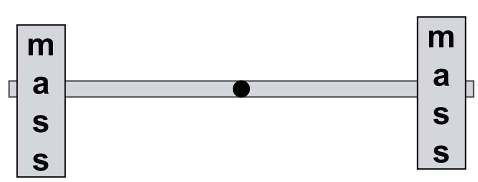
Distribution of Mass

When the mass is nearer the axis of rotation, it is easier to move, low rotational inertia.



When the mass is farther from the axis, it is harder to move, high rotational inertia.



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Conceptual Example

A person balancing on a tightrope or other tenuous object may put their arms outward to balance. They may even carry objects to help them stay balanced.

This extra mass far away from them increases their moment of inertia. This makes it more difficult for them to rotate, or tip over and fall.

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Moment of Inertia Formula

If you consider any object to be made up of a collection of particles

$$I = \Sigma m r^2$$

- I: Moment of inertia, $kg \bullet m^2$
- Σ: The sum of all particles
- m: Mass of particle, kg
- r^2: Radius from axis of rotation, m

This odd formula describes how difficult it is to make an object rotate.

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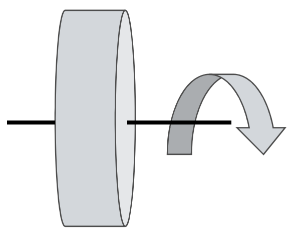
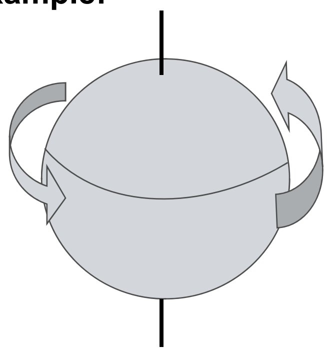
Other Shortcut Formulas

There are many formulas for finding the moment of inertia for various shapes. These can be found in any physics text. For example:





Solid disk

$I = 1/2\ mr^2$

Solid sphere

$I = 2/5\ mr^2$

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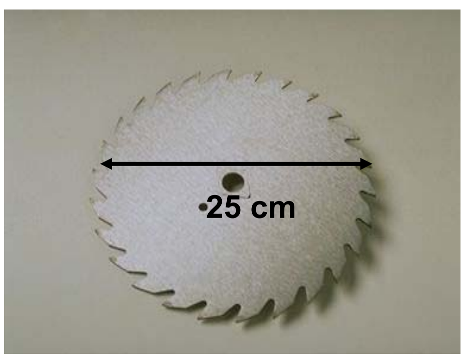
Moment of Inertia - Example

Calculate the moment of inertia of a saw blade. The blade has a diameter of 25 cm, and a mass of 0.70 kg.



Consider the blade a regular flat disk, neglecting the teeth and middle hole.

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Moment of Inertia - Solution

Change the given diameter into a radius and convert into meters:

$$25\text{cm} \div 2 = 12.5\text{cm} \times \frac{1\text{m}}{100\text{cm}} = .125\text{m}$$

Since we are considering the blade a solid disk, use the moment of inertia shortcut formula:

$$I = \frac{mr^2}{2}$$

$$I = \frac{(.7\text{kg})(.125\text{m})^2}{2} = .005\text{kgm}^2$$

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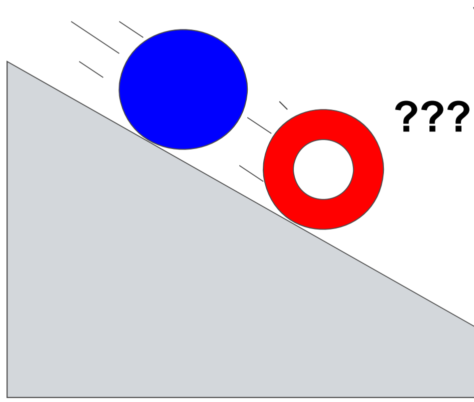
Mass and Inertia - Question

A disk, and a hoop have equal total masses. They are simultaneously released at the top of a hill. Which will hit the bottom first?

Hint: Consider where the mass is located on each.



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Mass and Inertia - Answer

Although they both have the same mass, thus the same pull from gravity, the hoop has a higher moment of inertia since its mass is located farther from its axis of rotation.

The disk has some of its mass nearer the axis of rotation, thus a lower moment of inertia.

A lower moment of inertia means it is easier for a torque to get the object rotating.

Thus, the disk will win the race every time!

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Rotational Motion and Equilibrium

Just as forces balance out to produce equilibrium, torques may cancel out to produce equilibrium too.

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Balancing Torques

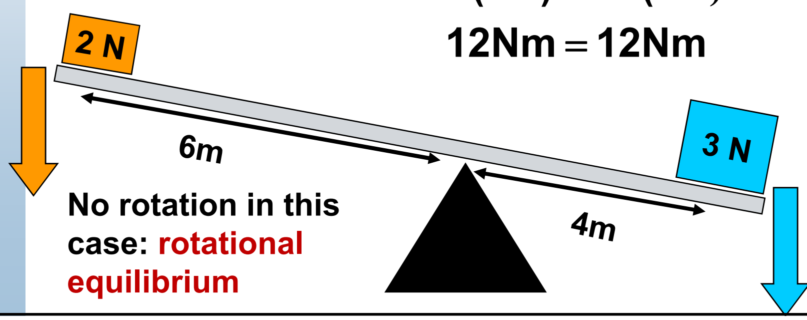
When the CW and CCW torques equal each other, then the object is in equilibrium. There is no rotation. Imagine this see-saw example shown below.

$$\tau_{\text{counterclockwise}} = \tau_{\text{clockwise}}$$

$$Fl = Fl$$

$$2N(6m) = 3N(4m)$$

$$12Nm = 12Nm$$



No rotation in this case: **rotational equilibrium**

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Rotational Equilibrium

If the torques do balance out, and there are no other net forces acting, the system would be in equilibrium.

Obviously the **forces** do not need to balance out, but the **torques** do. This is true because torque is the product of a force and a distance.

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Newton's Second Law Again

When torques aren't in equilibrium, rotation occurs. This is described by the rotational version of Newton's second law :

$$\Sigma\tau = I\alpha$$

The sum of the torques action on an object, N•m

Angular acceleration, rad/s^2

Moment of inertia, kg•m^2

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Torque Calculation

Using Newton's second law in rotational form, calculate the torque needed to accelerate the saw blade. Use the data from previous example problems.

$$I = .005\text{kgm}^2$$

$$\alpha = 51\text{rad/s}^2$$

$$\Sigma\tau = I\alpha$$

Newton's second law

$$\Sigma\tau = (.005\text{kgm}^2)(51\text{rad/s}^2)$$

Rad unit drops out

$$\Sigma\tau = .255\text{kgm}^2/\text{s}^2 = .255\text{ Nm}$$

1N = kg•m/s^2

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Angular Momentum

Your previous knowledge of linear momentum can be transferred to the angular arena.

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Angular Momentum Description

Just as an object moving in a straight line has linear momentum, an object moving in a circular path has angular momentum.

Angular momentum is a measure of the "strength" of an object's rotation about a particular axis.

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Angular Momentum Formula

Angular momentum is the product of two rotational quantities we learned earlier in this tutorial:

$$L = I\,\omega$$

- **Angular momentum, kg•m²/s** → L
- **Moment of inertia, kg•m²** → I
- **Angular speed, rad/s** → ω

Notice the very unusual unit for angular momentum! Calculations of angular momentum can become very cumbersome!

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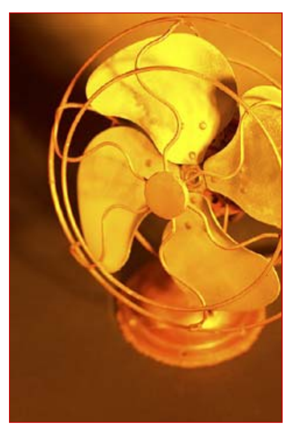
Similarities

The formula for linear momentum is analogous to the formula for angular momentum:

$$P = mv$$

Momentum ↕ mass ↕ velocity

$$L = I\omega$$



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Conservation of Angular Momentum

Just as linear momentum is conserved, so is angular momentum.

An object will maintain its angular momentum unless acted on by an unbalanced torque.

This is analogous to how an unbalanced force changes an object's linear momentum.

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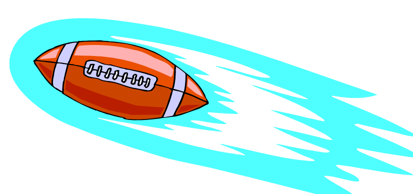
Stability

Often, objects are rotated so that they are more stable.

Because they have angular momentum, it is more difficult to change their motion.

This is why a football is thrown with a spiral motion.



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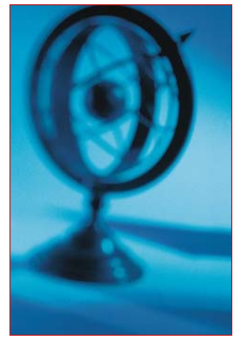
The Gyroscope

A gyroscope is simply a rotating object that has significant angular momentum.

Like a rotating football, that momentum is difficult to change, so the gyroscope is very stable.



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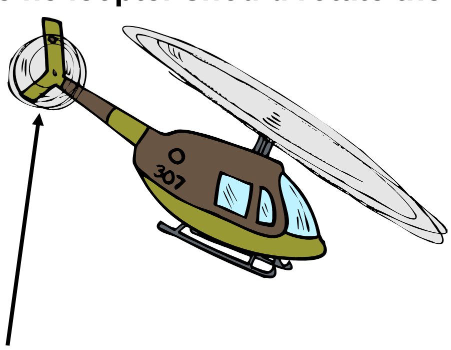
Helicopters and Momentum

Angular momentum is conserved by helicopters too. The main rotating blade has angular momentum. To keep momentum conserved, the body of the helicopter should rotate the opposite way!



The tail rotor counteracts this tendency and keeps the helicopter steady.

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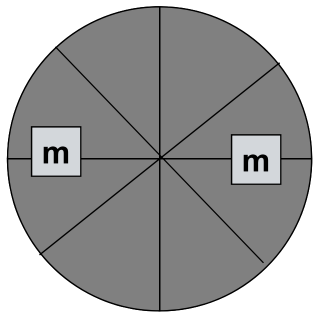
Momentum Concept Example

Imagine a rotating horizontal disk similar to a record player. It spins at some particular angular speed.



As it is spinning, additional masses are added to the disk. How will the situation change?

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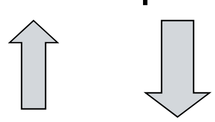
Changing Quantities

In the previous example, the angular speed obviously decreased.

This occurred because of the additional mass which raised the moment of inertia.

However, combining the two, shows that the angular momentum is conserved and stays constant.

angular momentum	moment of inertia	angular speed
constant	↑	↓



$$L = I\omega$$

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Learning Summary

The torque on an item equals the moment of inertia times the angular acceleration.

All linear kinematics equations can be translated into rotational varieties

All previous concepts involving linear momentum hold true for the angular momentum.

Torque is the product of force times lever arm.

Moment of inertia depends upon the mass of a item and its distribution of that mass.

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Congratulations

You have successfully completed the tutorial

Rotational Motion and Equilibrium

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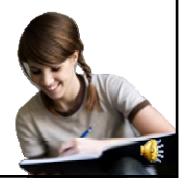
What's Next …

Step 1: Concepts – Core Tutorial (Just Completed)

→ **Step 2: Practice – Interactive Problem Drill**

Step 3: Recap – Super Review Cheat Sheet

Go for it!



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